Cebir I Final Cevap Anahtarı

1- a) $(a,p^2)=p \Rightarrow p|a \wedge p|p^2$ fakat $p^2 \nmid a$ dır.

$(a+p,\ p^2)=t$ olsun.

$p|a+p \wedge p|p^2 \Rightarrow p|t$ ① $t|a+p \wedge t|p^2 \Rightarrow t|p$

$\Rightarrow t|a+p \Rightarrow t|a$ olup $t|p$ bulunur. ②

b) $o(a)=n$ ve $o(bab^{-1})=t$ olsun.

$(bab^{-1})^t=e \Rightarrow (ba\cancel{b^{-1}})(\cancel{b}a\cancel{b^{-1}})\dots(\cancel{b}ab^{-1})=e$

$\Rightarrow ba^tb^{-1}=e \Rightarrow a^t=e$ olup $n|t$ ①

$o(a)=n \Rightarrow a^n=e \Rightarrow ba^nb^{-1}=e$

$\Rightarrow ba\dots ab^{-1}=e \Rightarrow bab^{-1}\dots bab^{-1}=e$ olur $t|n$ ②

$t=n$ bulunur.

2- $\mathbb{Z}_{15}^* = \{\bar{1},\bar{2},\bar{4},\bar{7},\bar{8},\overline{11},\overline{13},\overline{14}\}$ $H=\{\bar{1},\overline{14}\}$

$\cdot$	$\bar{1}$	$\overline{14}$
$\bar{1}$	$\bar{1}$	$\overline{14}$
$\overline{14}$	$\overline{14}$	$\bar{1}$

olup $H \leq \mathbb{Z}_{15}^*$ olur. $\mathbb{Z}_{15}^*$ değ. olduğundan

$H \trianglelefteq \mathbb{Z}_{15}^*$ bulunur.

$\bar{2}H=\{\bar{2},\overline{13}\}$

$\bar{4}H=\{\bar{4},\overline{11}\}$

$\bar{7}H=\{\bar{7},\bar{8}\}$

$\overline{11}H=\{\overline{11},\bar{4}\}$

$\overline{13}H=\{\overline{13},\bar{2}\}$

$\bar{8}H=\{\bar{8},\bar{7}\}$

$\bar{2}H=\overline{13}H$

$\bar{7}H=\bar{8}H$

$\mathbb{Z}_{15}^*/H=\{H,\ 2H,\ \bar{4}H,\ \bar{7}H\}$

3- Defterinizde var.

4-) $\mathbb{Z}_{12}^{*}=\{\bar{1},\bar{5},\bar{7},\bar{11}\}$

$f_{\bar{1}}=\begin{pmatrix}1&5&7&11\\1&5&7&11\end{pmatrix}$ ~~$f_{\bar{5}}=\begin{pmatrix}1&5&7&11\\5&7&11&1\end{pmatrix}$~~

$f_{\bar{5}}=\begin{pmatrix}1&5&7&11\\5&1&11&7\end{pmatrix}$ $f_{\bar{7}}=\begin{pmatrix}1&5&7&11\\7&11&1&5\end{pmatrix}$ $f_{\bar{11}}\begin{pmatrix}1&5&7&11\\11&7&5&1\end{pmatrix}$

$\varphi(\mathbb{Z}_{12}^{*})=\{\varphi(1),\varphi(5),\varphi(7),\varphi(11)\}$

	$\varphi(1)$	$\varphi(5)$	$\varphi(7)$	$\varphi(11)$
$\varphi(1)$	$\varphi(1)$	$\varphi(5)$	$\varphi(7)$	$\varphi(11)$
$\varphi(5)$	$\varphi(5)$	$\varphi(1)$	$\varphi(11)$	$\varphi(7)$
$\varphi(7)$	$\varphi(7)$	$\varphi(11)$	$\varphi(1)$	$\varphi(5)$
$\varphi(11)$	$\varphi(11)$	$\varphi(7)$	$\varphi(5)$	$\varphi(1)$

5) $a\in G$ için $\varphi_a: G\to G$, $x\to\varphi_a(x)=axa^{-1}$ iç otomorfizma olduğundan $I(G)\neq\emptyset$

$\forall a,b\in G$, $\varphi_a,\varphi_b\in I(G)$ için $\varphi_a\circ\varphi_b\in I(G)$ ve $\varphi_a^{-1}\in I(G)$ olduğunu göstermeliyiz.

$\forall x\in G$ için $(\varphi_a\circ\varphi_b)(x)=\varphi_a(\varphi_b(x))=a(bxb^{-1})a^{-1}$
$=abx(ab)^{-1}$
$=\varphi_{ab}(x)$

$\varphi_a\circ\varphi_b=\varphi_{ab}\in I(G)$ bulunur.

$\varphi_a\circ\varphi_{a^{-1}}=\varphi_{aa^{-1}}=\varphi_e$ ve $\varphi_{a^{-1}}\circ\varphi_a=\varphi_e$ dir.

$\forall x\in G$ için $\varphi_e(x)=x=I_G(x)$ olup φ_e, $O(G)$'nin birim elemanıdır. Buradan $\varphi_{a^{-1}}=\varphi_a^{-1}$ elde edilir.

O halde $I(G)\leq O(G)$ bulunur.