

06.04.2025

Adı- Soyadı:

Numara:

MAT204 ANALİTİK GEOMETRİ II DERSİ VİZE SINAVI

Soru 1) $\phi(x, y) = x^2 + 2xy + y^2 - 4\sqrt{2}y + 4 = 0$ koniğini

- i) Sınıflandırınız.
- ii) Merkezil hale getiriniz.

Soru 2) $\lambda x^2 + (\lambda - 2)xy + (\lambda - 2)y^2 + x - y = 0$ konik ailesindeki koniklerin sınıfını λ nın durumuna göre inceleyiniz.

Soru 3) $x^2 - xy - y^2 - 2x + 4y = 0$ koniği için

- i) Merkezini bulunuz.
- ii) $2x - y = 7$ doğrusuna paralel olan çap denklemini bulunuz.

Soru 4) $x^2 - xy + y^2 + x - y + 3 = 0$ koniğinin

- i) $x + 2y = 0$ doğrusuna dik olan çap denklemini bulunuz.
- ii) Eksen denklemlerini bulunuz.

Soru 5) $x^2 - 3xy + 6y^2 - 3x + 2y - 5 = 0$ koniğinin

- i) $A(0, -2)$ noktasından geçen çap denklemini bulunuz.
- ii) $A(1, -1)$ noktasını orta nokta kabul eden kirişin denklemini bulunuz.

Prof. Dr. Emin KASAP

$$\textcircled{1} \quad \phi(x,y) = x^2 + 2xy + y^2 - 4\sqrt{2}y + 4 = 0$$

$$\Delta = 4AC - B^2$$

$$4 \cdot 1 \cdot 1 - 2^2 = 0 \quad \text{parabol veya parabol yansıması}$$

$$K = \begin{vmatrix} 2A & B & D \\ B & 2C & E \\ D & E & 2F \end{vmatrix} \neq 0 \text{ halinde parabol.}$$

$$x^2 + 2xy + y^2 - 4\sqrt{2}y + 4 = 0 \quad \text{konjugat standart form getirelim.}$$

$$\begin{aligned} A &= 1 \\ B &= 2 \\ C &= 1 \end{aligned}$$

$$\tan 2\alpha = \frac{B}{A-C} = \frac{2}{0} = \alpha = \frac{\pi}{4}$$

$$x = \frac{\sqrt{2}}{2}(x' - y')$$

$$y = \frac{\sqrt{2}}{2}(x' + y')$$

$$\frac{1}{2}(x' - y')^2 + x'^2 - y'^2 + \frac{1}{2}(x' + y')^2 - 4x' - 4y' + 4 = 0$$

$$2x'^2 - 4x' - 4y' + 4 = 0$$

dir. Şimdi öteleme yapalım

$$x' = x'' + h$$

$$y' = y'' + k$$

$$2(x'' + h)^2 - 4(x'' + h) - 4y'' - 4k + 4 = 0$$

$$2x''^2 + 4x''h + 2h^2 - 4x'' - 4h - 4y'' - 4k + 4 = 0$$

$$2x''^2 + \underbrace{(4h - 4)}_0 x'' - 4y'' + \underbrace{2h^2 - 4h - 4k + 4}_0 = 0$$

$$\boxed{h=1}$$

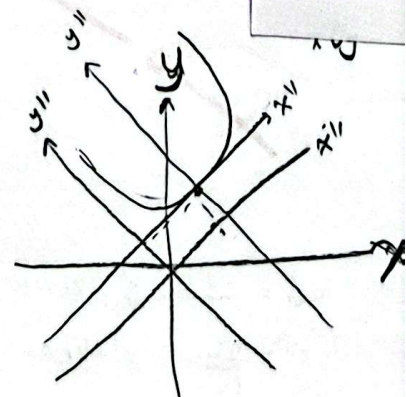
$$2 - 4 + 4 - 4k = 0$$

$$4k = 2 \quad \boxed{k = \frac{1}{2}}$$

$$2x''^2 - 4y'' = 0$$

$$x''^2 - 2y'' = 0$$

$$x''^2 = 2y'' \quad \text{Parabol} \Rightarrow y'' = \frac{1}{2}x''^2$$



$$\textcircled{1} \quad \phi(x,y) = x^2 + 2xy + y^2 - 4(2y+4) = 0$$

$$\Delta = 4AC - B^2$$

$$4 \cdot 1 \cdot 1 - 2^2 = 0 \quad \text{parabol veya parabol ya da doğru}$$

$$K = \begin{vmatrix} 2A & B & D \\ B & 2C & E \\ D & E & 2F \end{vmatrix} \neq 0 \text{ halinde parabol.}$$

$$2) \quad \lambda x^2 + (\lambda-2)xy + (\lambda-2)y^2 + x - y = 0$$

$$\Delta = 4AC - B^2 = 4\lambda(\lambda-2) - (\lambda-2)^2$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow \\ \lambda & \lambda-2 & (\lambda-2)^2 \end{matrix}$$

$$(\lambda-2)(4\lambda - \lambda + 2) = 0$$

$$(\lambda-2)(3\lambda + 2) = 0$$

$$\lambda_1 = 2 \quad \lambda_2 = -\frac{2}{3}$$

$$K = \begin{vmatrix} 2A & B & D \\ B & 2C & E \\ D & E & 2F \end{vmatrix} = \begin{vmatrix} 2\lambda & \lambda-2 & 1 \\ \lambda-2 & 2(\lambda-2) & -1 \\ 1 & -1 & 0 \end{vmatrix}$$

$$\begin{matrix} +2(\lambda-2) & 2\lambda & \lambda-2 & 1 & -\lambda+2 \\ 2\lambda & \lambda-2 & 2(\lambda-2) & -1 & -\lambda+2 \end{matrix}$$

$$-2\lambda+4 - 2\lambda+4 - 2\lambda = 0$$

$$-6\lambda+8=0$$

$$\boxed{\lambda_2 = +\frac{4}{3}}$$

	$-2/3$	$4/3$	2	
Δ	$+$ elips	$-$ hiperbol	$-$ hiperbol	$+$ elips
K	$+$ $K > 0$ sarı elips	$K \neq 0$ hiperbol $+$	$K \neq 0$ hiperbol $-$	$-$ $K < 0$ reel elips
	$\Delta = 0$ $K \neq 0$ parabol	$\Delta < 0$ $K = 0$ Kesişen iki doğru	$\Delta = 0$ $K \neq 0$ parabol	

3) $x^2 - xy - y^2 - 2x + 4y = 0$

a) $\phi_x = 2x - y - 2$

$\phi_y = -x - 2y + 4$

$M(x_0, y_0)$ merkez den

$\phi_x|_M = 0 \Rightarrow 2x_0 - y_0 = 2$

$\Rightarrow 5x_0 = 8$

$M\left(\frac{8}{5}, \frac{6}{5}\right)$

$\phi_y|_M = 0 \Rightarrow -x_0 - 2y_0 = -4$

$x_0 = \frac{8}{5}$

$y_0 = \frac{6}{5}$

b) $2x - y = 7$ eğimi 2 0 halde çapı eğimi $m' = 2$

$(m+m')B + 2Cmm' + 2A = 0$

$2 - 1 - 1 + 1 = 0$

$-m - 2 - 4m + 2 = 0 \Rightarrow -5m = 0 \Rightarrow m = 0$

0 halde çap denklemleri $\phi_x + m\phi_y = 0$ olup $\phi_x = 0$

yani

$\boxed{2x - y - 2 = 0}$

4) $x+2y=0$ eğimi $-\frac{1}{2}$. O halde $m' = +2$

i) $(m+m')B + 2Cmm' + 2A = 0$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $2 \quad -1 \quad 1 \quad 2 \quad 1$

$m = 0$

$\phi_x + 0 \cdot \phi_y = 0$

$\boxed{2x - y + 1 = 0}$

ii)

$m^2B + 2(A-C)m - B = 0$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $-1 \quad 1 \quad 1 \quad -1$

$-m^2 = 1 \Rightarrow m^2 = 1$
 $m = \pm 1$

$\phi_x + \phi_y = 0$

$\phi_x - \phi_y = 0$

$2x - y + 1 + (-x + 2y - 1) = 0$

$2x - y + 1 + x - 2y + 1 = 0$

$\boxed{x + y = 0}$

$\boxed{3x - 3y + 2 = 0}$

5) $x^2 - 3xy + 6y^2 - 3x + 2y - 5 = 0$

$\phi_x + m\phi_y = 0$

$(2x - 3y - 3) + m(-3x + 12y + 2) = 0 \Rightarrow 3 - 22m = 0$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $0 \quad -2 \quad 0 \quad -2$

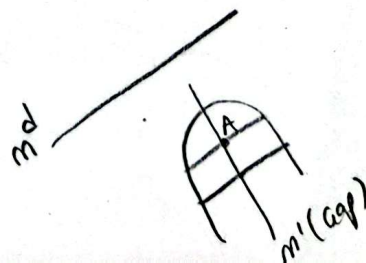
$22m = 3 \Rightarrow m = \frac{3}{22}$

Yani $\boxed{\phi_x + \frac{3}{22}\phi_y = 0}$

6) Orta nokta çap dairemizden geçer

$\phi_x + m\phi_y = 0$

$(2x - 3y - 3) + m(-3x + 12y + 2) = 0$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $1 \quad -1 \quad 1 \quad -1$



$$2 - 13m = 0$$

$$m = \frac{2}{13}$$

O halde kirisin egimi $\frac{2}{13}$.

O halde egimi ve
noktası bilinen doğru denklemi

$$y + 1 = \frac{2}{13} (x + 1)$$