MAT314 KOMP. FONK. TEO. GİRİŞ ARASINAV ÇÖZÜMLERİ

1) $|1-\bar{z}w|^2-|z-w|^2 = (1-\bar{z}w)\overline{(1-\bar{z}w)} - (z-w)\overline{(z-w)}$

$= (1-\bar{z}w)(1-z\bar{w}) - (z-w)(\bar{z}-\bar{w})$

$= 1-\cancel{z\bar{w}} - \bar{z}w + \bar{z}w z\bar{w} - z\bar{z} + \cancel{z\bar{w}} + w\bar{z} - w\bar{w}$

$= 1 + |z|^2|w|^2 - |z|^2 - |w|^2$

$= |z|^2(|w|^2-1) + 1 - |w|^2$

$= (1-|w|^2)(1-|z|^2)$

2) a) $\cos z = \dfrac{e^{z}+e^{-z}}{2} \Rightarrow$

$\cos(2+i) = \frac{1}{2}\left[e^{i(2+i)} + e^{i(-2-i)}\right] = \frac{1}{2}\left[e^{2i}\cdot e^{-1} + e^{-2i}\cdot e^{+1}\right]$

$= \frac{1}{2}\left[e^{-1}(\cos 2 + i\sin 2) + e^{1}\cdot(\cos 2 - i\sin 2)\right]$

$= \dfrac{e^{1}+e^{-1}}{2}\cdot\cos 2 + i\cdot\dfrac{e^{1}-e^{-1}}{2}\cdot\sin 2$

Dolayısıyla $\operatorname{Re} w = \dfrac{e^{1}+e^{-1}}{2}\cos 2$, $\operatorname{Im} w = \dfrac{e^{1}-e^{-1}}{2}\sin 2$

b) $z = e^{\overbrace{\left(\frac{2+\pi i}{4}\right)^2}^{u}} = e^{\left(\frac{4-\pi^2+4\pi i}{16}\right)} = \underbrace{e^{\overbrace{\frac{4-\pi^2}{16}+\frac{1}{4}\pi i}^{=u}}}$

$= e^{\frac{4-\pi^2}{16}}\cdot e^{\frac{\pi}{4}i} = e^{\frac{4-\pi^2}{16}}\cdot\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}\cdot e^{\frac{4-\pi^2}{16}} + i\frac{\sqrt{2}}{2}\cdot e^{\frac{4-\pi^2}{16}}$

$|z| = |e^{u}| = e^{\frac{4-\pi^2}{16}}$, $\arg z = \arg e^{u} = \frac{\pi}{4}$,

$\operatorname{Re} z = \frac{\sqrt{2}}{2}e^{\frac{4-\pi^2}{16}}$, $\operatorname{Im} z = \frac{\sqrt{2}}{2}e^{\frac{4-\pi^2}{16}}$

$z = e^{u} \Rightarrow |z| = |e^{u}| = e^{\operatorname{Re} u}$, $\arg e^{u} = \operatorname{Im} u$ idi.

③ a) $e^{z} = 1+i \Rightarrow z = \frac{1}{2}\left(\ln 2 + \frac{\pi}{2}\right) + 2k\pi i, \quad k \in \mathbb{Z}.$

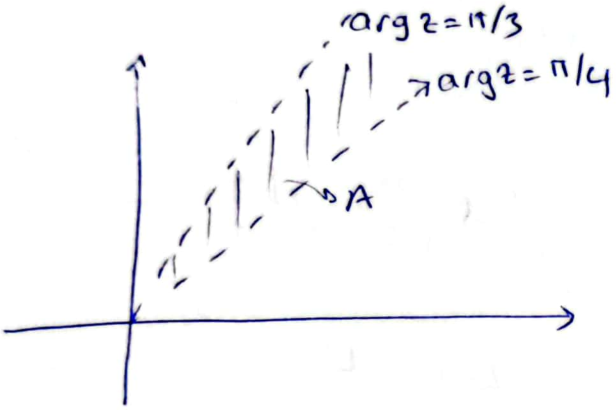
b) $z^{6} + i + 1 = 0 \Rightarrow z_k = \sqrt[6]{-1-i}$

$$= \sqrt[6]{\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)}$$

$k = 0,1,2,3,4.$

$$= \sqrt[12]{2}\cdot\left(\cos\frac{\frac{5\pi}{4}+2k\pi}{6} + i\sin\frac{\frac{5\pi}{4}+2k\pi}{6}\right)$$

④ $A = \{z \in \mathbb{C} : \frac{\pi}{4} < \arg z < \frac{\pi}{3}\}$



$\forall z \in A$ için $B(z,r) \subset A$ o.ş. $r>0$ var olduğundan $A^{\circ} = A$ dır. A açıktır.

Her $z \in \{z : \frac{\pi}{4} \leq z \leq \frac{\pi}{3}\}$ için $(B(z,r) - \{z\}) \cap A \neq \emptyset \quad \forall r>0$ olduğundan $A' = \{z : \frac{\pi}{4} \leq z \leq \frac{\pi}{3}\}$.

Her $z \in \{z : \arg z = \frac{\pi}{4}, \arg z = \frac{\pi}{3}\}$ için $B(z,r) \cap A \neq \emptyset$ ve $B(z,r) \cap (\mathbb{C} - A) \neq \emptyset$ olduğundan $\partial A = \{z : \arg z = \frac{\pi}{4}, \arg z = \frac{\pi}{3}\}$

⑤ a) $\cos z = \dfrac{e^{iz} + e^{-iz}}{2} = \dfrac{e^{-y+ix} + e^{y-ix}}{2}$

$$= \frac{1}{2}\left[e^{-y}(\cos x + i\sin x) + e^{y}(\cos x - i\sin x)\right]$$

$$= \cos x\left(\frac{e^{y}+e^{-y}}{2}\right) - i\sin x\left(\frac{e^{y}-e^{-y}}{2}\right)$$

$$= \cos x \cosh y - i \sin x \sinh y$$

b) $|\cos z|^{2} = \cos^{2}x\cosh^{2}y + \sin^{2}x\sinh^{2}y$

$$= \cos^{2}x(1+\sinh^{2}y) + (1-\cos^{2}x)\sinh^{2}y$$

$$= \cos^{2}x + \sinh^{2}y.$$