2023-2024 Eğitim Öğretim Yılı, MAT 311 Kısmi Diferansiyel Denklemler I, Final

Sıra no: Numara: İsim-Soyisim: **15/1/2024**

SORULAR

1. $xu_x+yu_y= xy(u^2+1)$ kısmi diferansiyel denkleminin genel çözümünü bulunuz.
2. $p^2-3pq+2q=6$ denkleminin tam integralini bulunuz.
3. $Z_{xx}-2Z_{xy}+Z_{yy}=0$ denklemini kanonik forma indirgeyiniz.
4. $U_t-2U_{xx}=0, U(t,0)=U(t,\pi)=0$ denkleminin genel çözümünü bulunuz.

Herbir soru 25 puandır. Sınav süresi 90 dakikadır. Süre bittikten sonra teslim edilen cevaplar kabul edilmeyecektir. Başarılar dilerim.

1) $a=x,\ b=y,\ c=xy(u^2+1) \rightarrow \frac{dx}{x}=\frac{dy}{y}=\frac{du}{xy(u^2+1)}$

$\frac{dx}{x}=\frac{dy}{y} \rightarrow \ln x=\ln y+\ln c_1=\ln yc_1 \rightarrow$ $x=yc_1 \rightarrow c_1=\frac{x}{y}=w$ 10

$\frac{dy}{y}=\frac{du}{xy(u^2+1)} \rightarrow dy=\frac{du}{c_1y(u^2+1)} \rightarrow \int c_1y\,dy=\int\frac{du}{u^2+1} \rightarrow \frac{c_1y^2}{2}=\arctan u+c_2$

$\frac{x}{y}\cdot\frac{y^2}{2}=\arctan u+c_2 \rightarrow$ $c_2=\frac{xy}{2}-\arctan u=v$ 10

$\text{grad}w\times\text{grad}v\neq 0 \rightarrow$ $F(w,v)=F\left(\frac{x}{y},\frac{xy}{2}-\arctan u\right)=0$ 5

2) $F(x,y,u,p,q)=p^2-3pq+2q-6=0$ 5

$\frac{dp}{F_x+pF_u}=\frac{du}{F_y+qF_u} \rightarrow$ $\frac{dp}{0}=\frac{dq}{0}$ 5 $\rightarrow dp=0 \rightarrow p=a \rightarrow a^2-6=3aq-2q$

$a^2-6=q(3a-2) \rightarrow \frac{a^2-6}{3a-2}=q \rightarrow du=p\,dx+q\,dy$ 15

$\rightarrow \int du=\int a\,dx+\int\frac{a^2-6}{3a-2}dy \rightarrow$ $u=ax+\frac{a^2-6}{3a-2}y+b$

3) $A=1,\ B=-2,\ C=1 \rightarrow \Delta=B^2-4AC=4-4=0$ parabolik

$\frac{dy}{dx}=\frac{B}{2A}=-1 \rightarrow dy=-2\,dx \rightarrow$ $y+2x=c_1=\xi$ 5, $\xi=y=c_2$ olsun 5

$z_x = z_\xi \xi_x + z_\eta \eta_x = z_\xi$

$z_{xx} = [z_{\xi\xi}\xi_x + z_{\xi\eta}\eta_x] = z_{\xi\xi}$

$z_{xy} = -[z_{\xi\xi}\xi_y + z_{\xi\eta}\eta_y] = -[z_{\xi\eta} + z_{\xi\eta}]$ 5

$z_y = z_\xi \xi_y + z_\eta \eta_y = z_\xi + z_\eta$

$z_{yy} = z_{\xi\xi}\xi_y + z_{\xi\eta}\eta_y + z_{\eta\xi}\xi_y + z_{\eta\eta}\eta_y = z_{\xi\xi} + 2z_{\xi\eta} + z_{\eta\eta}$

$z_{xx} - 2z_{xy} + z_{yy} = z_{\xi\xi} - 2z_{\xi\xi} - 2z_{\xi\eta} + z_{\xi\xi} + 2z_{\xi\eta} + z_{\eta\eta} = 0$

$z_{\eta\eta} = 0$ 10

4) $U_t - 2U_{xx} = 0$

$U(t,0) = U(t,\pi) = 0$

$U = XT \rightarrow \quad XT' - 2X''T = 0 \rightarrow \frac{XT'}{2XT} = \frac{X''T}{XT} = k \rightarrow X'' = kX$

$X(0) = X(\pi) = 0$

i) $k=0$ için $m^2 = 0,\ m_{1,2} = 0 \rightarrow X(x) = c_1 + c_2 x \rightarrow X(0) = 0 = c_1,\ X(\pi) = c_2\pi = 0,\ c_2 = 0$ 5

$X = 0$

ii) $k>0$ için $m^2 = k,\ m_{1,2} = \pm\sqrt{k} \rightarrow X(x) = c_1 e^{-\sqrt{k}x} + c_2 e^{\sqrt{k}x} \rightarrow X(0) = X(\pi) = 0 = c_1 = c_2$

$X = 0$ 5

iii) $k<0$ için $m^2 = k,\ m_{1,2} = \pm i\sqrt{k} \rightarrow X(x) = c_1 \cos\sqrt{k}x + c_2 \sin\sqrt{k}x$

$X(0) = 0 = c_1 \rightarrow X(\pi) = 0 = c_2 \sin\sqrt{k}\pi \rightarrow \sqrt{k}\pi = n\pi \rightarrow k = n^2 = k_n$

$X_n = c_n \sin nx$ 5

$T' = 2Tn^2 \rightarrow m = 2n^2 \rightarrow T(t) = Ae^{2n^2 t}$ 5

$U(x,t) = c_n \sin nx \cdot Ae^{2n^2 t}$

$U(x,t) = \sum_{n=1}^{\infty} c_n \sin nx \cdot Ae^{2n^2 t}$ 5