

# Sayılar Teorisi Dersi final cevap

## Anahtarları

$$1) x^2 + x + 48 \equiv 0 \pmod{125}$$

$$b_1 = 1, b_2 = 3$$

$$b_1 = 1 \quad f(1) = 50 \quad f'(1) = 3 \quad 3k \equiv -\frac{50}{5} \pmod{5}$$

$$k \equiv 0 \pmod{5} \quad x = b_1 + kp = 1 + 0 \pmod{25}$$

$$b_1' = 1 \quad f(1) = 50 \quad f'(1) = 3 \quad 3k \equiv -\frac{50}{25} \pmod{5}$$

$$k \equiv 1 \pmod{5} \quad x = b_1' + kp^2 = 1 + 25 = 26 //$$

$$b_2 = 3 \Rightarrow f(3) = 60 \quad f'(3) = 7 \quad 7k \equiv -\frac{60}{5} \pmod{5}$$

$$k \equiv 4 \pmod{5} \quad x = 3 + 20 = 23$$

$$b_2' = 23 \quad f(23) = 600 \quad f'(23) = 47$$

$$47k \equiv -\frac{600}{25} \pmod{5} \quad k = 3 \quad x = 23 + 75 \equiv \underline{\underline{98}} \pmod{125}$$

$$2-) x^{17} \equiv 23 \pmod{41} \quad (17, 40) = 1 \quad 36 \text{ üm vör.}$$

$$17 \text{ ind } x \equiv \text{ind } 23 \pmod{40}$$

$$17 \text{ ind } x \equiv 36 \pmod{40}$$

$$\text{ind } x = \underline{\underline{28}} \quad x = \underline{\underline{31}}$$

$$40 = 2 \cdot 17 + 6$$

$$17 = 2 \cdot 6 + 5$$

$$6 = 1 \cdot 5 + 1$$

$$1 = 3 \cdot 40 - 7 \cdot 17$$

$$36 = 108 \cdot 40 - 252 \cdot 17 \quad \text{ind } x = \underline{\underline{28}}$$

$$3-) a) \left( \frac{1091}{23} \right) = \left( \frac{16}{23} \right) = \left( \frac{2}{23} \right) \left( \frac{8}{23} \right) \equiv 1$$

$$\left( \frac{3}{13} \right) \left( \frac{13}{3} \right) = 1 \quad \left( \frac{23}{3} \right) = \left( \frac{1}{3} \right) = 1 \quad \underline{\underline{\text{İPTAL}}}$$

$$b) d = \frac{225}{157} \text{ bulunur.}$$

$$4-) a) \alpha = \frac{1+\sqrt{5}}{2} = 1 + \frac{1}{\frac{1+\sqrt{5}}{2}} = 1 + \frac{2}{1+\sqrt{5}} = 1 + \frac{\sqrt{5}-1}{2}$$

$$\frac{\sqrt{5}-1}{2} = 0 + \frac{1}{\frac{\sqrt{5}-1}{2}} = \frac{2}{\sqrt{5}-1} = \frac{2(\sqrt{5}+1)}{4} = \frac{\sqrt{5}+1}{2}$$

$\alpha = \{1, \bar{1}\}$  bulunur.

$$b) \alpha = 1 + \frac{1}{1 + \frac{1}{y}} = 1 + \frac{y}{y+1}$$

$$y = 1 + \frac{1}{2 + \frac{1}{y}} = \frac{3y+1}{2y+1} \quad 2y^2 + y = 3y + 1$$

$$2y^2 - 2y - 1 = 0 \quad y = \frac{1+\sqrt{3}}{2}$$

$$\alpha = 1 + \frac{\frac{1+\sqrt{3}}{2}}{\frac{1+\sqrt{3}}{2} + 1} = \frac{2\sqrt{3}+6}{6} = \frac{\sqrt{3}+3}{3}$$

$$5-) a) \sigma(p^2 q^2) = 1767 \text{ ise}$$

$$= \frac{p^3-1}{p-1} \cdot \frac{q^3-1}{q-1} = (p^2+p+1)(q^2+q+1)$$

$$= 31 \cdot 57$$

$$p=5, q=7$$

$$b) \sigma(p \cdot q) = 2pq$$

$$1+p+q+pq = 2pq \Rightarrow pq = 1+p+q$$

$$p = \frac{q+1}{q-1} \quad p=2 \text{ ise } q=3 \vee p=3 \text{ ise } q=2$$

olup  $n = 2 \cdot 3 = 6$  bulunur.