Adı Soyadı: **11.04.2025**

Numara:

1. soru	2. soru	3. soru	4. soru	5.soru	6.soru	7.soru	Toplam

MAT314 KOMPLEKS FONKSİYONLAR TEORİSİNE GİRİŞ

ARASINAV SORULARI

1. $\dfrac{(1+i)^7}{(1-i)^{10}}$ sayısını kutupsal formda yazınız.

   $\dfrac{(1+i)^7}{(1-i)^{10}}=\dfrac{1}{4}+\dfrac{i}{4}$ eşitliğini elde ediniz.

2. Aşağıda verilen deklemleri çözünüz. $(z\in\mathbb{C})$

   a) $e^z=-\sqrt{2}+\sqrt{2}i$

   b) $e^z=-2$

   c) $Log(z+1)=i\dfrac{\pi}{4}$

3. $(-1)^{1/\pi},(1-i)^{4i}$ ifadelerinin tüm değerlerini bulunuz.

4. $z^4+z^2+1=0$ $(z\in\mathbb{C})$ denkleminin köklerini bulunuz.

5. Eşlenik ve modül özelliklerin kullanarak $\left|(2\overline{z}+5)(\sqrt{2}-i)\right|=\sqrt{3}\left|2z+5\right|$ olduğunu gösteriniz. $A=\left\{z\in\mathbb{C}:\left|2\overline{z}+i\right|=4\right\}$ kümesini belirtip çiziniz.

6. $w=\left(\dfrac{1+i}{\sqrt{2}}\right)^{2i}$ kompleks sayısı verilsin. $|w|,\operatorname{Re}w,\arg w,\operatorname{Im}w$ değerlerini bulunuz.

7. $iz^2+(1-5i)z-1+8i=0$ denklemini çözünüz, köklerini $a+ib$ şeklinde yazınız.

Not: 1. soru 10 puan, diğer sorular 15'er puandır. Süre 100 dakikadır.

Başarılar...

Prof. Dr. Birsen SAĞIR DUYAR

11.04.2025

MAT314 Komp. Fonk. Teo. Giriş Arasınav Çözümleri

① $1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right) \Rightarrow (1+i)^7 = 2^{7/2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$

$(1-i)=2^{1/2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right) \Rightarrow (1-i)^{10} = 2^5\left(\cos\left(\frac{-10\pi}{4}\right)+i\sin\left(\frac{-10\pi}{4}\right)\right)$

$$\frac{(1+i)^7}{(1-i)^{10}} = \frac{2^{7/2}\left[\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right]}{2^5\left[\cos\left(-\frac{10\pi}{4}\right)+i\sin\left(-\frac{10\pi}{4}\right)\right]}$$

$$=2^{-3/2}\left[\cos\frac{17\pi}{4}+i\sin\frac{17\pi}{4}\right] = 2^{-3/2}\left[\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

$$=2^{-3/2}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right) = \frac{1}{4}+\frac{i}{4}$$

② a) $z=\log(-\sqrt{2}+\sqrt{2}i) = \ln|-\sqrt{2}+\sqrt{2}i| + i\arg(-\sqrt{2}+\sqrt{2}i)$

$= \ln 2 + i\left(\frac{3\pi}{4}+2k\pi\right), \quad k\in\mathbb{Z}$

b) $e^z=-2 \Rightarrow e^x(\cos y+i\sin y) = -2+i0 \Rightarrow e^x\cos y + ie^x\sin y = -2+i0$

$e^x\cos y=-2, \quad e^x\sin y = 0$ olur.

$\Downarrow$

$e^x>0$ olduğundan $\sin y=0 \Rightarrow y=k\pi, k\in\mathbb{Z}$.

Bu değer ilk denklemde yerine yazılırsa

$e^x\cos y=-2 \Leftrightarrow e^x\cos(k\pi)=-2 \Leftrightarrow e^x(-1)^k=-2$

ifadesinin sağlanabilmesi için $k=2m+1, \ m\in\mathbb{Z}$ olmalı.

$\Rightarrow e^x=2 \quad \Rightarrow x=\ln 2$

Çözüm kümesi $= \{(x,y) : x=\ln 2, \ y=(2m+1)\pi, \ m\in\mathbb{Z}\}$

c) $\log(z+1) = \ln|z+1| + i\arg(z+1) = i\frac{\pi}{4} \Rightarrow$

$\ln|z+1|=0, \ \arg(z+1)=\frac{\pi}{4} \quad \left(\arg(z+1) = \arctan\frac{y}{x+1} = \frac{\pi}{4}\right)$

$\Rightarrow (x+1)^2+y^2=1, \quad \frac{y}{x+1}=\tan\frac{\pi}{4}=1 \quad \Rightarrow x=\frac{1}{\sqrt{2}}-1, \ y=\frac{1}{\sqrt{2}}$

bulunur. $z=\frac{1}{\sqrt{2}}-1+i\frac{1}{\sqrt{2}}$ olur.

③ $(-1)^{1/\pi}$, $(1-i)^{4i}$ ifadelerinin tüm değerlerini bulunuz.

$(-1)^{1/\pi} = e^{\frac{1}{\pi}\log(-1)} = e^{\frac{1}{\pi}[\ln|-1| + i\arg(-1)]} = e^{\frac{1}{\pi} i\cdot(\pi+2k\pi)}$

$= e^{(2k+1)i}$, $k\in\mathbb{Z}$.

$(1-i)^{4i} = e^{4i\log(1-i)} = e^{4i(\ln|1-i| + i(-\frac{\pi}{4}+2k\pi))}$

$= e^{4i[\ln\sqrt{2} + i\arg(1-i)]} = e^{i\ln 4 - 4(-\frac{\pi}{4}+2k\pi)}$

$= e^{-4(-\frac{\pi}{4}+2k\pi)}\cdot[\cos(\ln 4) + i\sin\ln 4]$, $k\in\mathbb{Z}$

④ $z^4+z^2+1=0$, $z^2=u$ olsun. $u^2+u+1=0$ olur.

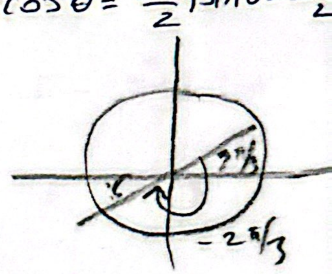
$\Delta = b^2-4ac = 1-4.1.1 = -3$ $\quad u_{1,2} = \frac{-1\mp\sqrt{-3}}{2} = \frac{-1\mp i\sqrt{3}}{2}$

$u_1 = \frac{-1+i\sqrt{3}}{2}$, $u_2=\frac{-1-i\sqrt{3}}{2}$, $z^2 = \frac{-1+i\sqrt{3}}{2}$ olsun.

$z = \left(\frac{-1+i\sqrt{3}}{2}\right)^{1/2}$, $w = \frac{-1+i\sqrt{3}}{2}$, $|w|=1$, $\text{Arg}\,w=\theta$, $\cos\theta = \frac{-1}{2}$, $\sin\theta=\frac{\sqrt{3}}{2}$

$\text{Arg}\,w = \theta = \frac{2\pi}{3}$,

$z_k = 1\cdot\left[\cos\frac{\frac{2\pi}{3}+2k\pi}{2} + i\sin\frac{\frac{2\pi}{3}+2k\pi}{2}\right)$ $\quad k=0,1$



$= \cos\left(\frac{\pi}{3}+k\pi\right) + i\sin\left(\frac{\pi}{3}+k\pi\right)$

$z_0 = \cos\frac{\pi}{3}+i\sin\frac{\pi}{3} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$, $z_1 = \cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$z^2=u = \frac{-1-i\sqrt{3}}{2} \Rightarrow z = \left(\frac{-1-i\sqrt{3}}{2}\right)^{1/2}$, $w = \frac{-1-i\sqrt{3}}{2}$, $|w|=1$,

$\text{Arg}\,w=\theta$, $\cos\theta = -\frac{1}{2}$, $\sin\theta = -\frac{\sqrt{3}}{2}$, $\theta = -\frac{2\pi}{3} = -\pi+\frac{\pi}{3}$

$z_k = 1.\left(\cos\frac{-\frac{2\pi}{3}+2k\pi}{2} + i\sin\frac{-\frac{2\pi}{3}+2k\pi}{2}\right)$, $k=0,1$

$= \cos\left(-\frac{\pi}{3}+k\pi\right) + i\sin\left(-\frac{\pi}{3}+k\pi\right)$, $k=0,1$

$\boxed{z_0 = \cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right) = \frac{1}{2} - i\frac{\sqrt{3}}{2}}$

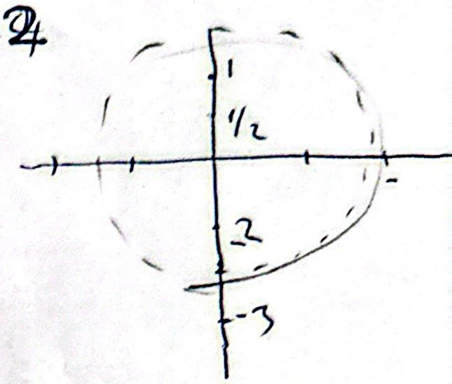
$\boxed{z_1 = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}}$

(5) $|(2\bar{z}+5)(\sqrt{2}-i)| = |2\bar{z}+5|\,|\sqrt{2}-i|$

$= |\overline{2z+5}|\cdot\sqrt{2+1} = \sqrt{3}\cdot|2z+5|$

$|2\bar{z}+i| = 4 \Rightarrow 2\cdot\left|\bar{z}+\frac{i}{2}\right| = 4 \Rightarrow \left|z-\frac{i}{2}\right| = 2$

$M(0, 1/2)$, $r=2$ olan çember.



(6) $w = \left(\frac{1+i}{\sqrt{2}}\right)^{2i} = e^{2i\log\left(\frac{1+i}{\sqrt{2}}\right)}$

$= e^{2i\left[\ln\left|\frac{1+i}{\sqrt{2}}\right| + i\arg\left(\frac{1+i}{\sqrt{2}}\right)\right]} = e^{2i\left[\ln 1 + i\left(\frac{\pi}{4}+2k\pi\right)\right]}$

$= e^{-2\left(\frac{\pi}{4}+2k\pi\right)}, \quad k\in\mathbb{Z}$

$|w| = e^{-2\left(\frac{\pi}{4}+2k\pi\right)}$, $\operatorname{Re} w = e^{-2\left(\frac{\pi}{4}+2k\pi\right)}$, $\operatorname{Im} w = 0$,

$\operatorname{Arg} w = 0$, $\arg w = 2k\pi$, $k\in\mathbb{Z}$

(7) $\Delta = b^2-4ac = (2+5i)^2 - 4\cdot 2\cdot(i-2) = 4+20i-25+8i\mp16$

$\Delta = 12i-5$, $\quad z_{1,2} = \dfrac{2+5i\mp\sqrt{12i-5}}{4}$

$\sqrt{12i-5} = (12i-5)^{1/2}$, $w = 12i-5$, $|w| = 13$

$\mp\sqrt{\dfrac{a+\sqrt{a^2+b^2}}{2}} + \mu\cdot i\cdot\sqrt{\dfrac{-a+\sqrt{a^2+b^2}}{2}}$

$= \mp\sqrt{\dfrac{-5+\sqrt{144+25}}{2}} + i\sqrt{\dfrac{5+\sqrt{144+25}}{2}} = \mp\sqrt{\dfrac{-5+13}{2}} + i\sqrt{\dfrac{5+13}{2}}$

$= \mp(2+i3)$

$z_1 = \dfrac{2+5i+2+3i}{4} = \dfrac{4+8i}{4} \Rightarrow \boxed{z_1 = 1+2i}$

$z_2 = \dfrac{2+5i-2-3i}{4} = \dfrac{0+2i}{4} \Rightarrow \boxed{z_2 = \frac{1}{2}i}$