Cevap Anahtarı

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MAT 302 DİFERENSİYEL GEOMETRİ II KISA SINAV SORULARI

SORU: $\alpha(t)=\left(t\,,\,\frac{1-t}{t},\frac{1-t^2}{t}\right)$ parametrik denklemi ile verilen α eğrisinin t=1 noktasında T,N,B Frenet vektörlerini, κ eğriliğini ve τ burulmasını hesaplayınız.

Başarılar
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Not: Süre 45 dakikadır.

MAT 302 Diferansiyel Geometri II Kısa Sınav

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SORU: $\alpha(t) = \left(t, \dfrac{1-t}{t}, \dfrac{1-t^2}{t}\right)$ $t=1$ nok $T, N, B, \kappa, \tau = ?$

$$\alpha'(t) = \left(1, -\frac{1}{t^2}, \frac{-t^2-1}{t^2}\right)$$

$$\alpha''(t) = \left(0, \frac{2}{t^3}, \frac{2}{t^3}\right)$$

$$\alpha'(1) = \left(1, -1, \frac{-2}{1}\right) = (1, -1, -2)$$

$$\|\alpha'(1)\| = \sqrt{1+1+4} = \sqrt{6} \neq 1$$

birim hızlı değil

$$T(1) = \frac{\alpha'(1)}{\|\alpha'(1)\|} = \boxed{\frac{1}{\sqrt{6}}(1, -1, -2)}$$

$$\alpha''(1) = (0, 2, 2)$$

$$B(1) = \frac{\alpha'(1) \otimes \alpha''(1)}{\|\alpha'(1) \otimes \alpha''(1)\|}$$

$$\alpha'(1) \otimes \alpha''(1) = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & -1 & -2 \\ 0 & 2 & 2 \end{vmatrix} = (2, -2, 2)$$

$$\|\alpha'(1) \otimes \alpha''(1)\| = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$$

$$B(1) = \boxed{\frac{1}{\sqrt{3}}(1, -1, 1)}$$

$$N = B \otimes T = \begin{vmatrix} e_1 & e_2 & e_3 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \end{vmatrix} = \boxed{\left(\frac{3}{\underset{9.2}{\sqrt{18}}}, \frac{+3}{\sqrt{18}}, 0\right)}$$

$$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

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$$\kappa(1) = \frac{\|\alpha'(1) \otimes \alpha''(1)\|}{\|\alpha'(1)\|^3} \qquad \tau(1) = \frac{\det(\alpha'(1), \alpha''(1), \alpha'''(1))}{\|\alpha'(1) \otimes \alpha''(1)\|^2}$$

$$\kappa(1) = \frac{2\sqrt{3}}{(\sqrt{6})^3} = \frac{2\sqrt{3}}{(\sqrt{6})^2 \cdot \sqrt{6}}$$

$$\alpha'''(t) = (0, -6, -6)$$

$$= \frac{\overset{1}{\cancel{2}}\sqrt{3}}{\underset{3}{\cancel{6}} \cdot \sqrt{6}} = \frac{1}{\sqrt{\underset{9 \cdot 2}{18}}} = \frac{1}{3\sqrt{2}}$$

$$\det(\alpha'(1), \alpha''(1), \alpha'''(1)) = \begin{vmatrix} 1 & -1 & -2 \\ 0 & 2 & 2 \\ 0 & -6 & -6 \end{vmatrix} = 0$$

$$\tau(1) = 0$$